Final Exam Practice Answers

Multiple Choice

1. C
2. A
3. E
4. A

5. C (Note that this question uses a slightly different convention – from our class notes, you should have obtained -4 for your answer. Just make it positive and take the square root.)

6. C
7. C
8. A
9. A
10. D
11. C
12. B
13. D
14. B
15. B
16. A
The $PV$ diagram for a heat engine is shown above. The cycle consists of two constant volume
and two constant pressure processes. The pressure and volume at point $a$ are $P_a = 3.0 \times 10^5 \text{ Pa}$ and
$V_a = 6.0 \times 10^{-3} \text{ m}^3$. The pressure and volume at point $c$ are $P_c = 6.0 \times 10^5 \text{ Pa}$ and $V_c = 2.0 \times 10^{-3} \text{ m}^3$. Assume that the working substance of this engine is one mole of an ideal gas with $c_v = \frac{5}{2} R$ and $c_p = \frac{3}{2} R$.

(a) Find the total work done by the engine.

The easiest way to find the work done is to find the area enclosed by the cycle. Since the cycle is a
rectangle, just multiply the width by the height, recognizing that $P_b = P_a$ and $V_d = V_a$.

$$W_{net} = (P_c - P_b)(V_d - V_c) = (P_c - P_a)(V_a - V_c) = (3 \times 10^5)(4 \times 10^{-3}) \text{ J}$$

$$W_{net} = 1200 \text{ J}$$

(b) Find the heat transferred into the gas for $b \rightarrow c$ and $c \rightarrow d$ — this is the “energy in” that we
will use to calculate the efficiency. (Hint: Use the first law of thermodynamics.)

To find the heat transferred to the gas, we’ll need to know some of the temperatures. The ideal gas
law says that $PV = nRT$, so state $d$, which has the highest pressure and volume, will have the highest
temperature. Similarly, state $b$ has the lowest temperature. During this cycle, the gas absorbs heat
from $b$ to $c$ and from $c$ to $d$ and then expels heat from $d$ to $a$ and $a$ to $b$. So the heat transferred to
the gas is $Q_{in} = Q_{bc} + Q_{cd}$. 

2
We need $T_b, T_c$ and $T_d$.

\[
T_b = \frac{P_b V_b}{R} = 72.167 \text{ K}
\]
\[
T_c = \frac{P_c V_c}{R} = 144.33 \text{ K}
\]
\[
T_d = \frac{P_d V_d}{R} = 433 \text{ K}
\]

Process $b \rightarrow c$ occurs at constant volume so the gas does no work. From the First Law, we know that the heat transferred into the gas will be the same as the change in internal energy of the gas.

\[
Q_{bc} = \Delta E_{\text{int}} \\
= nC_v \Delta T_{bc} \\
= \frac{3}{2} R(T_c - T_b) \\
= \frac{3}{2} (8.314)(144.33 - 72.167) \text{ J}
\]

\[
Q_{bc} = 900 \text{ J}
\]

Process $c \rightarrow d$ occurs at constant pressure. We could calculate the work done and the change in internal energy to determine the heat transfer. However, since we have the specific heat at constant pressure, $C_p$, we can more simply calculate

\[
Q_{cd} = nC_p \Delta T_{cd} \\
= \frac{5}{2} R(T_d - T_c) \\
= \frac{5}{2} (8.314)(433 - 144.33) \text{ J}
\]

\[
Q_{cd} = 6000 \text{ J}
\]

The total heat transferred is $Q_{in} = Q_{bc} + Q_{cd} = 6900 \text{ J}$.

(c) What is the total entropy change for this cycle? Explain.

If these are reversible processes, then the total entropy change is zero. If the processes are irreversible, then the total entropy change will be greater than zero. Since the gas returns to the same state every cycle, the entropy change of the gas will be zero. However, the entire system, which is composed of the gas and the hot and cold reservoirs, may have an entropy increase.

(d) What is the efficiency of this engine (the efficiency is the ratio of the “work out” to the “energy in”).

The efficiency is

\[
\epsilon = \frac{W_{\text{net}}}{Q_{in}} \\
= \frac{1200}{6900} \\
\epsilon = 17.4\%
\]
(e) How does this compare to the efficiency of a Carnot cycle operating between the same temperature extremes?

The Carnot efficiency between the same temperature extremes is

$$
\epsilon_c = 1 - \frac{T_L}{T_H}
$$

$$
= 1 - \frac{72.167}{433}
$$

$$
\epsilon_c = 82.3\%
$$

As expected, this is a much greater efficiency than actually obtained, as it represents the theoretical maximum for any cycle operating between these two temperature extremes.