1. Goals of this lab

You will learn how Maple help to perform operations on vector functions and to solve related problems, as well as plotting simultaneously diverse objects such as space curves, intersections of surfaces, planar projections and tangent lines.

2. Group work and Lab Report presentation

I strongly recommend to:

- Type comments on your Maple work sheet to explain and organize your work
- Enter restart: and needed packages at the beginning of each problem to empty Maple internal memory and to download appropriate packages.
- Frequently save your work (after each maple operation).
- See Guide to Lab posted at the course web page, as well as the previous labs for more on Maple commands and presentation.

- Work with your partners and be sure that you give the opportunity to each member of the group to participate (establish a rotation for example).
- Work with your partners and be sure that you give the opportunity to each member of the group to participate (establish a rotation for example). Each Lab group should turn in only one report for grading. NUMERICAL and LABEL all answers and all Graphs. DO NOT HAND IN UNNECESSARY WORK! Where appropriate, you must explain your reasoning in text or mathematically.

- You can email me your report. Your Lab report is supposed to be well organized and with all the explanations needed. The lab-report will be graded on its quality and its presentation (10 points).

3. Maple commands needed:

The documents with the Labs #1 & #2 took you through most (but not all) of the MAPLE commands that you need. So, have them handy with you (or get them from the homework web page). You can always use the Maple help menu to get most of Maple commands, in particular for all their related OPTIONS. Of course, you can use either the Maple mode or the 2-dimensional Math mode. In addition, use the shortcuts from the icons on the tablet at the left of the Maple window worksheet as well as the right click to display the available options.

In order to help you work faster and get the most of Maple, below are new commands with some from labs #1 & #2. In particular, a selection of new Maple commands needed to do all standard vector function operations and plotting. You work under both Maple mode and Math mode.

3.1. Vector function operations.

- A vector function is in the form $r(t) = \langle x(t), y(t), z(t) \rangle$.
- A vector function is written in Maple with either Square Brackets or angle brackets: $r(t) := \langle x(t), y(t), z(t) \rangle$ or $r(t) := \{ x(t), y(t), z(t) \}$. Since we are using operations with vectors (with angle brackets), it is recommended to always use angle brackets to allow more flexibility of operations.
- Its graph is a space curve with a parameter $t$, so you have to use the spacecurve command for $t$ between any two numbers for the domain.
- Example with the space curve with equation $r(t) = \langle \cos t, \sin t, t \rangle$.

restart; 
with plots: 

$r(t) := \langle \cos t, \sin t, t \rangle$; 

spacecurve($r(t), t = 0..8*Pi, axes = box$); 

curve := spacecurve($r(t), t = 0..8*Pi$); 
display (curve, labels = $[x, y, z]$); 
display (curve, thickness = 5, color = green); 
tubeplot ($r(t), t = -5..10, radius = 0.2$); 
diff ($r(t), t$); 
$dr(t) := diff (r(t), t)$; 
$dr(1) := subs (t = 1, dr(t))$; 
evall(%) ;

You can also use the function option: $f(x)$ then just type $f(1)$ to get the value of $f$ at 1.

To use the function option, just enter the function, click enter and select that this is a function when you are asked.
3.2. Other useful operations.  
Right click or Left Tablet  Then select the operation or the option
\[ p := (1 + x)^2 * (x - 3); \quad \text{Let } p = (1 + x)^2(x - 3) \]
\[ \text{expand}(p); \quad \text{Expand } p. \]
\[ \text{evalf}(p); \quad \text{Write } p \text{ as floating point number.} \]
\[ \text{factor}(p); \quad \text{Factor } p. \]
\[ \text{solve}(p = 0, x); \quad \text{Solve } p = 0 \text{ for } x \text{ symbolically.} \]
\[ \text{fsolve}(p = 0, x); \quad \text{Solve } p = 0 \text{ for } x \text{ numerically.} \]

4. Initialization:

Start your lab work on Maple by entering the following (T means text, and > means either Math of Maple Mode):

| T | Lab 3: Vector Functions and Space Curves |
| T | Date |
| T | Group members |
| > | restart; |
| > | with(plots): |
| T | Problem 1 |

If needed, Load the packages \( \text{with(LinearAlgebra)} \) and \( \text{with(VectorCalculus)} \) like in lab #2 to perform some vector operations.

5. Lab requirements

Note: 2 points for the quality of presentation of your work. Email me your Maple file.

**Problem 1.** (5 points) For similar examples, see your notes in class, the attached Maple file, and/or Example 5 section 13.1.

Consider the cylinder \( x^2 + y^2 = 4 \) and the surface \( z = xy \).

(1) Plot the two surfaces in the same system of axis, each with different color.
(2) Describe, intuitively, their intersection.
(3) Find parametric equations (or vector functions) for the two curves that form this intersection.
(4) Finally, simultaneously plot these curves with the two surfaces to confirm your answer.

**IMPORTANT:** For the space curves you may need to use the options thickness and color to distinguish between the two curves and to make them visible on the surfaces, use the option \( \text{style=wireframe} \) to make the surfaces transparent, and you may use more options for the surfaces using the icons on the tabular.

**Problem 2.** (5 points) For similar examples, see your notes in class, the attached Maple file, and/or Example 6 section 13.1.

Consider the space curve \( x = \cos t, \quad y = \sin t, \quad z = \sin^2 t \).

(1) Without plotting this curve, show that this curve lies on the cylinder \( x^2 + y^2 = 1 \).
(2) Plot this curve (without the cylinder), then use the appropriate rotations to see the planar projections on the \( xy \)-plane, the \( xz \)-plane and the \( yz \)-plane.
(3) Now, find the equations of each of the 3 planar projections, plot them and compare to your work on question 2 to confirm your answers.

**Problem 3.** (5 points) Consider the vector function \( r(t) = \langle \cos t, 3e^{2t}, 3e^{-2t} \rangle \). Find parametric equations of the tangent line to the corresponding curve at the point \( P(1, 3, 3) \). Illustrate by graphing both the curve and the tangent line on common (be careful with the scale!).

**Problem 4.** (8 points) Let \( r(t) = \langle \cos t, \sin t, \frac{t^2}{2} + 1 \rangle \).
(1) At any time \( t \), compute the tangent and the unit tangent vectors, the velocity, the speed and the acceleration vector.
(2) Then, at \( P(1, 0, 1) \), compute the velocity, the speed, the vectors \( T, \ N \) and \( B \), the acceleration vector and its tangential and normal components, and the the curvature \( k \). Hint: find the value of \( t \) that correspond to that point \( P(1, 0, 1) \).
(3) At the same point \( P(1, 0, 1) \), find equations of the tangent line, and the plane that contains the vectors \( T \) and \( N \).
(4) At the same point \( P(1, 0, 1) \), find the radius, center and the equation of the osculating circle.
(5) Finally, using Maple, give the best possible visual illustrations of all these objects found in questions 2, 3 and 4 in the figure.