1. Goals of this lab

You will learn how Maple can help to perform vector operations, to plot and generate equations of arrows representing vectors, lines and planes, and to solve related problems.

2. Group Work and Lab Report Presentation

- Work with your partners and be sure that you give the opportunity to each member of the group to participate (establish a rotation for example). Each Lab group should turn in only one report for grading. NUMBER and LABEL all answers and all Graphs. DO NOT HAND IN UNNECESSARY WORK! Where appropriate, you must explain your reasoning in text or mathematically.

- It is not acceptable to just staple together a bunch of math formulas and some computer graphics printouts. Your Lab report is supposed to be well organized and with all the explanations needed. The lab-report will be graded on its quality and its presentation (20 points). Each problem is worth 20 points. See Guide to Lab at my web site.

- In order to easily save your work and include any comments, I strongly recommend that your work should regularly be saved as a Maple-file and/or as a Word-file, and share it with all members of your group.

3. Reminder of Some Mathematical Formulas

One application of vector operations involves the computation of the equations of lines and planes.

- A line that passes through the point \( P_0(x_0, y_0, z_0) \) and whose direction is that of the vector \( v = \langle a, b, c \rangle \), has the equation

\[
\begin{aligned}
\text{in Parametric form:} & \quad x = at + x_0, \quad y = bt + y_0, \quad z = ct + z_0. \\
\text{in Vector form:} & \quad \mathbf{r} = \overrightarrow{OP}_0 + tv, \quad \text{where} \quad \mathbf{r} = \langle x, y, z \rangle.
\end{aligned}
\]

- A plane that passes through the point \( P_0(x_0, y_0, z_0) \) and whose normal vector in \( \mathbf{n} = \langle a, b, c \rangle \), has the equation

\[
\begin{aligned}
\text{in Cartesian form:} & \quad a(x - x_0) + b(y - y_0) + c(z - z_0) = 0 \\
\text{in Vector form} & \quad \mathbf{n} \cdot (\mathbf{r} - \overrightarrow{OP}_0), \quad \text{where} \quad \mathbf{r} = \langle x, y, z \rangle.
\end{aligned}
\]

4. Some of Maple Commands that you need:

The Lab #1 took you through most (but not all) of the MAPLE commands. So, before starting each of the problems assigned below, it is important that you look at the lab #1 to see commands that you may need and the options available in Maple Math mode. Lab #1 is also available online at the homework webpage. In particular, review the following in the Worksheet Mode:

- the difference between \texttt{plot3d} (for functions) and \texttt{implicitplot3d} for equations.
- plotting a surface using \texttt{plot3d} or \texttt{implicitplot3d} while including options such as color, domain, axes, labels, title, etc.
- Using \texttt{:=} to define and give name to mathematical object in order to simplify your work.
• In Lab#1 review how to plot different figures simultaneously using the := to define each object, and then display commands to display all the objects simultaneously.

• In Lab #1, review how to plot a vector starting at a given point using the arrow command.

In order to help you work faster and get the most of Maple, below are new commands with some from lab#1. In particular, a selection of new Maple commands needed to do all standard vector operations using the “LinearAlgebra” package and others for plotting vectors, lines and planes using the “plots” package.

4.1. Plot a plane.

For example, how to plot a plane with equation \(2x + 3y + 4z = 5\)? It is an implicit equation, so you have to use the implicitplot3d command. For example:

\[
\text{implicitplot3d}(2\text{iz} + 3\text{y} + 4\text{z} = 5, \text{x} = -4..4, \text{y} = -3..5, \text{z} = -3..4, \text{axes} = \text{frame}).
\]

Of course you can change the options depending on the equations and the questions.

4.2. Plot a line.

How to plot a line with equation \(l(t) = (1 + 2t, 2 - 3t, 4 + 2t)\)? It is a space curve with a parameter \(t\), so you have to use the spacecurve command for \(t\) chosen between any two numbers. For example:

\[
\text{spacecurve}([1 + 2\text{t}, 2 - 3\text{t}, 4 + 2\text{t}], \text{t} = -4..5, \text{axes} = \text{normal}, \text{thickness} = 5, \text{color} = \text{green}, \text{axes} = \text{frame}, \text{labels} = \{\text{x, y, z}\}).
\]

Again, you can choose different options and you must choose a reasonable domain for \(t\) to have a good plot.

4.3. Vector operations.

You can choose the Vector/Matrices icon on the left for the Math Mode, or enter the following as Maple commands (under >) as follows:

\[
\text{with(LinearAlgebra):} \quad \text{Load Maple’s LinearAlgebra package.}
\]

\[
< a, b, c >; \quad \text{displayed as a column when the components are separated by commas.}
\]

\[
< a | b | c >; \quad \text{displayed as a row when separated by vertical bars (to save space).}
\]

\[
u := < 1, 2, 3 >; \quad \text{Let } u := < 1, 2, 3 >; \text{ using the symbol ”:=” to assign a notation/name.}
\]

\[
v := < 4, 5, 6 >; \quad \text{Let } v := < 4, 5, 6 > \text{ (or use the row display notation).}
\]

\[
w := u + v; \quad \text{Let } w \text{ be the sum of the vectors } u \text{ and } v.
\]

\[
3 \ast u; \quad \text{Multiply the vectors } u \text{ by 3 (YOU NEED * for Maple multiplication.}
\]

\[
2 \ast u - v; \quad \text{perform the operation.}
\]

\[
u \cdot v; \quad \text{Compute the dot product } u \cdot v.
\]

\[
u \times v; \quad \text{Compute the cross product } u \times v.
\]

\[
\text{Norm}(u, 2); \quad \text{Compute the magnitude of the vector } u.
\]

\[
\text{VectorAngle}(u, v); \quad \text{Compute the angle between the vectors } u \text{ and } v \text{ (in Radians)}
\]

\[
evalf(\%); \quad \text{Give an evaluation of the previous expression.}
\]

4.4. Plotting vectors.

\[
\text{with(plots):} \quad \text{Load Maple’s Plots package.}
\]

\[
\text{arrow}(u); \quad \text{plot the vector } u \text{ as arrow with its base located at the origin}
\]

\[
P := < 3, 0, 2 >; \quad \text{Let } P \text{ be the point } (3, 0, 2).
\]

\[
\text{arrow}(P, u) \quad \text{plot the vector } u \text{ with its base located at the point } P.
\]

\[
\text{arrow}(P, u, \text{options}) \quad \text{add more options to the figure (see below).}
\]

Example of options:

\[
\text{arrow}(P, u, \text{width} = 0.2, \text{headlength} = 0.4, \text{color} = \text{green}, \text{axes} = \text{boxedlabels}[x, y, z])
\]
5. Initialization:

Start your lab work on Maple by entering the following; where > means you need to enter it as maple command by clicking on the icon, and T means you need to enter it as a text by clicking on the command:

<table>
<thead>
<tr>
<th>T</th>
<th>Lab 2: Vectors, lines and Planes in the space</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>Enter the lab tile (as text).</td>
</tr>
<tr>
<td>T</td>
<td>Friday 02/08/2008</td>
</tr>
<tr>
<td>T</td>
<td>Enter today's date (as text).</td>
</tr>
<tr>
<td>&gt;</td>
<td>Group members</td>
</tr>
<tr>
<td>&gt;</td>
<td>to clear Maple's internal memory</td>
</tr>
<tr>
<td>&gt;</td>
<td>with(plots):</td>
</tr>
<tr>
<td>&gt;</td>
<td>Load Maple's Plots package.</td>
</tr>
<tr>
<td>&gt;</td>
<td>with(LinearAlgebra):</td>
</tr>
<tr>
<td>T</td>
<td>Load Maple's LinearAlgebra package.</td>
</tr>
</tbody>
</table>

6. Lab Report Requirements

Problem 1. Consider the point A(2,6,-1), B(-1,4,2), C(2,2,7), and F(0,6,5).

(1) Find the volume of the parallelepiped with edges $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AF}$.

(2) Plot these three vectors, each in a different color, using the arrow command.

Problem 2. Consider the points P(3,1,2), Q(1,1,4) and R(3,-1,4).

(1) Show that these three points are the vertices of an equilateral triangle by computing the lengths of the three sides.

(2) Find the vector/parametric equations of the lines that form the sides of this triangle.

(3) Finally, using the display command, plot simultaneously these 3 lines (using the space-curve) and arrows representing the sides of the triangle (using arrow command). Each line should be in a different color and each arrow should match the color of its line. Choose an appropriate domain for $t$ and rotate the figure so you can clearly see the equilateral triangle.

Problem 3. Consider the line $L(t) = (-3 - 3t, 4 + t, 11 + 6t)$ and the plane $P: 2x - 3y + 4z = 1$.

(1) Plot them simultaneously on the same figure using display command (you need to make a smart choices for the domains of $t$ for the line and for the domains of $x, y$ and $z$ for the plane).

(2) Find the angle between the line and the normal vector to the plane and determine if the line and the plane are parallel or perpendicular or neither.

(3) If they intersect, find their point of intersection. (HINT: The line $x = -3 - 3t, y = 4 + t, z = 11 + 6t$ should satisfy the equation of the plane, solve for $t$ and finally substitute $t$ by its value in the equation of the line to find the point of intersection.)

Problem 4. Consider the planes $P_1: x + y - z = 1$ and $P_2: 2x - 3y + 4z = 5$. For hints: see examples in the textbook section 12.5 and in the Notes on lines and planes posted on the homework webpage.

(1) Find the parametric equations for the line $L$ of intersection of the planes $P_1: x + y - z = 1$ and $P_2: 2x - 3y + 4z = 5$. (HINT: Put the two equations of the planes together, solve $x$ and $y$ as functions of $z$, then let $z = t$).

(2) Finally, plot them (the line and the planes) simultaneously on the same figure. Use a different color of each of the objects.