1. GOALS OF THIS LAB

You will learn how to use Maple to compute Partial Derivatives of functions of several variables and to plot simultaneously related objects: Surface graph, directional curves in this surface in the \( x \) or \( y \) directions (intersections of the graph with vertical planes in the \( x \) or \( y \) directions), Tangent Lines in a given direction, Tangent planes, Linear approximation, meaning of the partial derivatives, and to solve related problems.

2. GROUP WORK AND LAB REPORT PRESENTATION

- NUMBER and LABEL all answers and all Graphs. DO NOT HAND IN UNNECESSARY WORK!
Where appropriate, you must explain your reasoning in text or mathematically.
- The lab-report will be graded on its quality and its presentation. See Guide to Lab at my web site and comment in previous labs.
- Work with your partners and be sure that you give the opportunity to each member of the group to participate (establish a rotation for example). Each Lab group should turn in only one report for grading.
- In order to easily save and present your work and Maple internal memory, I strongly recommend that you type in comments and frequently save your work as a Word-file or Maple-file, and after each problem re-enter restart: to empty Maple internal memory, and >with(plots).

3. MAPLE COMMANDS THAT YOU NEED:

Review all previous labs for the commands that you need. So, have them handy with you or get them from the homework web page.

4. INITIALIZATION:

For MAPLE, enter >restart; then >with(plots).

5. REMINDER: SOME USEFUL FORMULAS FROM CHAPTER 14

- The equation of the tangent plane to the surface \( z = f(x, y) \) at the point \( P(a, b, c) \) where \( c = f(a, b) \) is:
  \[
  z = f_x(a, b) (x - a) + f_y(a, b) (y - b) + c
  \]

- The linear approximation of \( f \) near \( (a, b) \) is the equation of the tangent plane at the point \( P(a, b, c) \) where \( c = f(a, b) \) is:
  \[
  z = L(x, y) = f_x(a, b) (x - a) + f_y(a, b) (y - b) + c
  \]

- The tangent line in the \( x \) direction at the point \( P(a, b, c) \) is the tangent at \( P(a, b, c) \) of the curve obtained by the intersection of the surface-graph of \( f(x, y) \) and the vertical plane \( y = b \) (see example 2 page 913). Moreover, for this tangent line we have:
  1. a corresponding equation in the plane \( y = b \) is \( z = c + f_x(a, b) (x - a) \)
  2. a direction vector in the space of this tangent line is given by: \( \langle 1, 0, f_x(a, b) \rangle \)
  3. a corresponding parametric equations in the space: \( \langle t + a, b, f_x(a, b) t + c \rangle \)

- Similarly for the tangent line in the \( y \) direction at \( P(a, b, c) \). In particular, a direction vector of this tangent line is given by: \( \langle 0, 1, f_y(a, b) \rangle \) and parametric equations: \( \langle t + a, b, f_y(a, b) t + c \rangle \)

- Review Chapter 13 for parametric equations of a line, the intersection of two surfaces and its parametric equations.
- Go over example 2 from section 14.3 (page 913) and the extra explanations below.
- Read sections 14.3 and 14.4 for more examples.

- **How to find the parametric equations of the curve (parabola C1) of the intersection of the surface** \( z = 4 - x^2 - 2y^2 \) **and the vertical plane** \( y = 1 \)?
  Since it is a space curve, the parametric equation is a vector function \( r(t) = \langle x(t), y(t), z(t) \rangle \).
  Since \( y = 1 \) (we are at this plane) then we have \( y(t) = 1 \).
  In the other hand, if we substitute \( y = 1 \) in the equation of the surface, we get: \( z = 4 - x^2 - 21^2 \) or \( z = 2 - x^2 \). Therefore if we put \( x = t \) we then have \( z = 2 - t^2 \). So \( r(t) = (t, 1, 2 - t^2) \).

- **How to find the equation of the tangent line in the \( x \) direction at the point \( x = 1 \) and \( y = 1 \)?**
  The vector function of this line has the form \( L_x(t) = \langle x(t), y(t), z(t) \rangle \). Since we are dealing with the \( x \)-direction at \( x = 1 \) and \( y = 1 \), then \( y = \text{constant} = 1 \). So no change in \( y \), and the slope of this tangent line is the partial derivative \( f_x(1, 1) = -2 \), in the \( x \)-direction. In particular, a direction vector is \( \langle 1, 0, -2 \rangle \). Since the line passes through \( (1,1,1) \), a parametric equation is \( L_x(t) = \langle 1 \cdot t + 1, 0 \cdot t + 1, -2t + 1 \rangle = (t + 1, 1, -2t + 1) \).

- **Use similar method for the \( y \) direction.**

7. Problems to do

**Problem 1.** **Goal: The tangent plane at a point \( P \) resembles to the surface near that point \( P \).**

We are interested in the function \( z = f(x, y) = \sqrt{x - y} \) near the point \( P(5,1,2) \). Graph the surface and the tangent plane at the point \( P \). Then zoom in until the surface and the tangent plane become indistinguishable. In Maple you zoom-in by changing the domain of \( x \) and \( y \) (in the command) to a smaller one and closer to \( x = 5 \) and \( y = 1 \). You should include 3 plots that show that the more you zoom in, the flatter the graph appears and the more it resembles its tangent plane. See Example 1 section 14.4 for help.

**Problem 2.** **Goal: The linear approximation is a good local approximation to the values of \( f(x,y) \).**

Find the linear approximation \( L(x,y) \) to the function \( f(x,y) = \ln(x - 3y) \) at \((7,2)\), and use it to approximate \( f(6.9,1.08) \). Illustrate by graphing \( f \) and the tangent plane. For help, read Examples 2 and 3 from section 14.4 (pages 926-927).

**Problem 3.** **Goal: The tangent lines and the curves of intersection in the \( x \) and \( y \) directions**

The paraboloid \( z = 6 - x - x^2 - 2y^2 \) intersects the the vertical plane \( x = 1 \) in a parabola. Find parametric equations for the tangent line to this parabola at the point \( P(1,2,-4) \). Use Maple to graph the paraboloid, the parabola, and the tangent line on the same screen. You are asked to do something similar to the Example 2 from section 14.3 (pages 913); read this example and the ”extra explanations” about it in the first page.

**Problem 4.** **Goal: Abstract formula, the meaning the partial derivatives, and the heat equation**

In a study of frost penetration it was found that the temperature \( T \) at time \( t \) at a depth \( x \) can be modeled by the function:

\[
T(x,t) = T_0 + T_1 e^{-\lambda x} \sin(\omega t - \lambda x) \text{ T=temperature, t= time in days, x=depth in feet} \omega = \frac{2\pi}{365} \text{ and } \lambda > 0.
\]

1. Find \( T_x = \frac{\partial T}{\partial x} \). What is its physical significance?
2. Find \( T_t = \frac{\partial T}{\partial t} \). What is its physical significance?
3. Show that \( T \) satisfies the heat equation \( T_t = kT_{xx} \) for a certain constant \( k \).
4. If \( \lambda = 0.2 \), \( T_0 = 0 \), and \( T_1 = 10 \), use Maple to graph \( T(x,y) \). Be sure to choose the domain wisely.