3 Max-Min Problems

The typical max-min problems requires finding either the absolute maximum value or the absolute minimum value of a function, \( f(x) \), on an interval \( a \leq x \leq b \). Our first step is plot the graph. We want the \( x \)-values to contain the interval \([a, b]\) but not much larger. The \( y \)-values need to scaled in such a way that we get a clear picture of the largest and smallest values. If the function takes on negative values then the absolute minimum will be the largest negative value (the lowest point on the graph in any case).

Using either the crosshair for precise approximations or just using the labels on the \( y \)-axis to obtain a rough answer are both good first steps to solving the problem. To obtain exact answers we need calculus and critical point theory. In the Algebra Window we use the \[ \text{ } \] button to calculate \( f'(x) \). Next we use the \[ \text{ } \] button to find the critical points from the equation \( f'(x) = 0 \). Once that value is found it can be substituted back into the expression for \( f(x) \) by pressing the \[ \text{ } \] button.

Three problems arise:

- First we are only interested in those points \( x \) which lie on our interval \([a, b]\). It probably will be necessary to obtain decimal approximations to our critical points \( x \) to see if they satisfy \( a \leq x \leq b \).

- The second problem is that the absolute maximum may not occur at the critical points at all. Instead, it occurs at either the left or right endpoint, i.e., \( x = a \) or \( x = b \). This situation should be clear from the graph.

- The third problem is that perhaps DERIVE can not solve the equation \( f'(x) = 0 \). As you look at the graph, you should be able to guess the critical point, at least approximately. Then you can ask DERIVE to approximate the critical point by using the Solve/Numerically \(^1\) menu. You will need to specify an interval on which DERIVE will search for a solution. But this should be obtained by again looking the graph and choosing a convenient interval containing the approximate critical point. Choosing too big of an interval can get you into trouble because DERIVE may end up finding a solution which is not the one you want. You would then need to refine your interval to exclude the undesired critical points.