Problem 1. You are at the point \((-1, 2, 3)\) facing the \(xz\)-plane. You walk 3 units forward, turn left and walk for an other 3 units. What is your final position?

From \((-1, 2, 3)\) we move 3 units in the \(-j\) direction to get to: \((-1, -1, 3)\).
From \((-1, -1, 3)\) we move 3 units in the \(i\) direction to get to: \((2, -1, 3)\).

Problem 2. Suppose \(a\) and \(b\) are vector that lie in the plane \(3x + 2y - 5z = 1\). Compute

\[(2a + 3b) \cdot (3, 2, -5)\]

It is obvious that the vector \((2a + 3b)\) also lies in the given plane. Therefore \(n\) is orthogonal to all vectors in that plane. In particular \(n\) is orthogonal to the vector \((2a + 3b)\). Thus \((2a + 3b) \cdot (3, 2, -5) = 0\).

Problem 3. Describe the surface whose equation in cylindrical coordinates is \(z = 4r\).

Using cylindrical coordinates, we have \(r^2 = x^2 + y^2\). Therefore \(z^2 = (4r)^2 = 4^2(x^2 + y^2)\). Thus \(\frac{z^2}{4^2} = x^2 + y^2\), which is the equation of a cone, but since \(z = 4r \leq 0\), it is just the half-cone opening upward with axis the \(z\)-axis and vertex \((0, 0, 0)\).

Problem 4. Describe (in words) and find the equation of the intersection of the surface \(x = y^2 + z^2\) and the plane \(\text{mathbf{x}} = y\).

The intersection occurs when \(x = y\) and \(x = y^2 + z^2\). By substitution we have: \(y = y^2 + z^2\), that we can write: \(y^2 - y + z^2 = 0\). By completing the square, we get: \((y - \frac{1}{2})^2 + z^2 = \left(\frac{1}{2}\right)^2\), which is the equation of a circle in the plane \(x = y\) with radius \(\frac{1}{2}\) and center \(\left(\frac{1}{2}, \frac{1}{2}, 0\right)\). (For the center \(x = \frac{1}{2}\) because \(x = y\).

Problem 5. Find the area of quadrilateral ABCD where \(A(2, 5, 0)\), \(B(1, 1, 0)\), \(C(8, 7, 0)\) and \(D(6, 2, 0)\). Note that ABCD is not a parallelogram (Draw a picture in the \(xy\)-plane).

Recall that the area of a triangle determined by the vectors \(u\) and \(v\) is half the area of the parallelogram determined these two vectors, and therefore equal to \(\frac{1}{2} |u \times v|\).

\[
\text{Area of quadrilateral ABCD} = \frac{1}{2} \text{Area of triangle ABD} + \frac{1}{2} \text{Area of triangle CBD}
\]

\[
= \frac{1}{2} |\overrightarrow{AB} \times \overrightarrow{AD}| + \frac{1}{2} |\overrightarrow{CB} \times \overrightarrow{CD}|
\]

\[
= \frac{1}{2} |(1, 4, 0) \times (5, 1, 0)| + \frac{1}{2} |(-6, -2, 0) \times (-2, -5, 0)|
\]

\[
= \frac{1}{2} |10| + \frac{1}{2} |26k|
\]

\[
= \frac{45}{2} = 22.5
\]

Problem 6. Consider the two planes \(2x - 3y + 5z = 2\) and \(4x + y - 3z = 7\).

1. Find the angle between the two planes.

2. Find the equation of the plane perpendicular to these two planes and that passes through the point \((4, 5, 6)\).
1.) The angle between the two planes is the angle between the normal vectors of these two planes. A normal vector to the first plane is \( \vec{n}_1 = (2, -3, 5) \). A normal vector to the second plane is \( \vec{n}_2 = (4, 1, -3) \). Let \( \theta \) be the angle between \( \vec{n}_1 \) and \( \vec{n}_2 \). We have

\[
\cos \theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1| |\vec{n}_2|} = \frac{-10}{\sqrt{38} \sqrt{26}}. \quad \text{Thus} \quad \theta \simeq 108.5^\circ \simeq 1.89\text{rad}
\]

(You can also use the cross product, but the dot product gives better result because with the cross product you find \( |\sin \theta| \) not \( \sin \theta \).

2.) A normal vector \( \vec{n} \) to this plane must be perpendicular to the normal vectors \( \vec{n}_1 \) and \( \vec{n}_2 \). Therefore \( \vec{n} = \vec{n}_1 \times \vec{n}_2 = 4i + 26j + 14k \). Thus, an equation of this plane that passes through \( (4, 5, 6) \) is:

\[
4(x - 4) + 26(y - 5) + 14(z - 6) = 0, \quad \text{in other terms:} \quad 1x + 26y + 14z = 230.
\]

**Problem 7.** A bird is flying with the velocity \( \vec{v} = 10i + 2j \) relative to the air (the speed is measured in m/sec, \( i \) represents the unit vector heading West and \( j \) is the unit vector heading North). The wind is blowing from the west at the speed of 5 m/sec.

1. Draw a picture showing the vectors:
2. Find the components of the vectors \( \vec{w} \) and \( \vec{r} \).
3. Find the speed of the bird relative to the ground.

\[
\vec{v} = \text{the velocity of the bird relative to the air} = 10i + 2j.
\]

\[
\vec{W} = \text{the velocity of the wind} = -5i.
\]

\[
\vec{u} = \text{the velocity of the bird relative to the ground} = \vec{v} + \vec{W} = 10i + 2j - 5i = 5i + 2j.
\]

Therefore, the ground speed is \( |\vec{u}| = \sqrt{25 + 4} = \sqrt{29} \).

**Problem 8.** Consider the two lines: \( L_1: x = -t, y = t, z = 2t \) and \( L_2: x = 3 + s, y = 3s, z = 5 - 4s \).

1. Show that these two lines are not parallel.
2. Show that these two lines do not intersect.
3. Find then the distance between these (skew) lines \( L_1 \) and \( L_2 \).

1.) A direction vector for the line \( L_1 \) is \( \vec{v}_1 = (-1, 1, 2) \), and a direction vector for the line \( L_2 \) is \( \vec{v}_2 = (1, 3, -4) \). These two vectors are not parallel because it is not possible to write \( \vec{v}_1 = c \vec{v}_2 \) for some real \( c \), or because their cross product is not zero: \( \vec{v}_1 \times \vec{v}_2 = -10i - 2j - 4k \neq 0 \). Therefore the two lines are not parallel.

2.) For the intersection, we solve for \( t \) and \( s \) the following equations:

\[
\text{(same } x\text{)} - t = 3 + s \quad , \quad \text{(same } y\text{)} t = 3s \quad , \quad \text{(same } z\text{)} 2t = 5 - 4s
\]

Solving the first two equations, we get \( s = -\frac{3}{4} \) and \( t = -\frac{9}{4} \). But these value do not satisfy the third equation (by substitution we get \( -\frac{9}{2} = 8 \) which is impossible). Thus \( L_1 \) and \( L_2 \) do not intersect.

3.) Since the two lines are skew, they can be viewed as lying on two parallel planes, and then the distance is the distance between one of the planes to a point in the other plane. A normal vector \( \vec{n} \) to these planes must be perpendicular to \( \vec{v}_1 \) and \( \vec{v}_2 \) (direction vectors of the two lines). So \( \vec{n} = \vec{v}_1 \times \vec{v}_2 = -10i - 2j - 4k \).

We set \( t = 0 \) in the equation for \( L_1 \), we get the point \( (0, 0, 0) \), then the equation of the plane that contains \( L_1 \) is:

\[
-10x - 2y - 4z = 0 \quad \text{or} \quad 10x + 2y + 4z = 0
\]

If we now set \( s = 0 \) in the equation for \( L_2 \), we get the point \( (3, 0, 5) \). So the distance between \( L_1 \) and \( L_2 \) is equal to the distance between this point and the plane which is given by the formula:

\[
D = \frac{|10(3) + 2(0) + 4(5)|}{\sqrt{10^2 + 2^2 + 4^2}} = \frac{50}{\sqrt{120}} \simeq 4.56
\]