Section 4.8 : Applications to Economics

Homework assignments

Go over examples 1, 2 & 3 in section 4.8 of the textbook
Section 4.8 : 1, 3, 5, 11, 12, 15, 19, 21, 23
The problems below

Definitions and Notations

1. **Cost function** \( C(x) \): The Cost of producing \( x \) units of a certain product.

2. **Marginal Cost** \( C'(x) \): The derivative of \( C(x) \); The cost of producing one more unit after \( x \) units have been produced.

3. **Average Cost** \( c(x) = \frac{C(x)}{x} \): The cost per unit when \( x \) units are produced.

4. **Price or Demand function** \( p(x) \): The price per unit that the company can charge if it sells \( x \) units.

5. **Revenue** \( R(x) = \text{Quantity} \times \text{Price} \): The Revenue from selling \( x \) units.

6. **Marginal Revenue** \( R'(x) \): The derivative of \( R(x) \); the Revenue from selling one more unit after \( x \) units have been sold.

7. **Profit** \( \Pi(x) = R(x) - C(x) \)

**Problem 1.** It is estimated that the cost of constructing an office building that is \( n \) floors high is \( C(n) = 2n^2 + 500n + 800 \) thousand dollars. How many floors should the building have in order to minimize the average cost per floor? (20 floors)

**Problem 2.** Find the quantity \( q \) of items which maximizes the profit if it is not possible to produce more than 800 items, and if the total revenue and the total cost (in dollars) are given by

\[ R(q) = 50q - 0.03q^2 \quad \text{and} \quad C(q) = 3000 + 20q \]

(500)

**Problem 3.** A baseball team plays in a stadium that hold 55,000 spectators. With ticket prices at $10, the average attendance had been 27,000. A market survey showed that for each $0.10 decrease in the ticket prices, on the average, the attendance will increase by 300. How should ticket prices be set to maximize revenue? ($9.50)

**Problem 4.** The regular air fare between Boston and San Francisco is $500. An airline using planes with a capacity of 300 passengers on this route observes that they fly with an average of 180 passengers. Market research tells the airlines’ managers that each $5 fare reduction would attract, on average, 3 more passengers for each flight. How should they set the fare to maximize their revenue? Explain your reasoning to receive credit. ($400)
**Solution of Problem 1:**
The cost $C(n)$ is given as a function of the number of floors $n$. Therefore the average cost is given by $c(n) = \frac{C(n)}{n} = \frac{2n^2 + 500n + 800}{n} = 2n + 500 + \frac{800}{n}$. We must then minimize the average cost. We need first to find the critical numbers:

$C'(n) = 2 + 0 - \frac{800}{n^2} = 0 \Rightarrow 2 = \frac{800}{n^2} \Rightarrow 2n^2 = 800 \Rightarrow n^2 = 400 \Rightarrow n = 20$ (because $n$ positive, thus it is the only critical number).

$R(n) = (2 - \frac{800}{n^2})' = -800(x^{-2})' = 1,600x^{-3} \Rightarrow R''(20) = 1,600(20)^{-3} > 0$. By the second derivative test, $R$ has a local minimum at $n = 20$, which is an absolute minimum since it is the only critical number.

The number of floors that will minimize the average cost per floor is then: $n = 20$.

**Solution of Problem 2:**
Let $n$ = number of times the price of the ticket is reduced by $0.10$ dollars, then:

\[
\begin{align*}
\text{price} & = 10 - n \cdot (0.10) = 10 - 0.10n \\
\text{quantity} & = \text{number of spectators} = 27,000 + 300n \\
\end{align*}
\]

Hence $R(n) = (27,000 + 300n)(10 - 0.10n) = 270,000 + 300n - 30n^2$ to maximize.

$R'(n) = 300 - 60n = 0 \Rightarrow n = \frac{300}{60} = 5$ is the only critical number.

$R''(n) = -60 \Rightarrow R''(5) = -60 < 0$. By the second derivative test, $R$ has a local maximum at $n = 5$, which is an absolute maximum since it is the only critical number.

The best ticket prices to maximize the revenue is then: $\$10 - 0.10(5) = 9.50$, with $27,000 + 300(5) = 28,500$ spectators and a revenue of $\$270,750$.

**Solution of Problem 3:**
Let $R$ = the revenue function = quantity $\times$ price

Let $n$ = number of times the fare is reduced by $\$5$ dollars, then:

\[
\begin{align*}
\text{price} & = 500 - n \cdot (5) = 500 - 5n \\
\text{quantity} & = \text{number of passengers} = 180 + 3n \\
\end{align*}
\]

Hence $R(n) = (180 + 3n)(500 - 5n) = 90,000 + 600n - 15n^2$ to maximize (for $0 \leq n \leq 40$).

$R'(n) = 600 - 30n = 0 \Rightarrow n = \frac{600}{30} = 20$ is the only critical number.

$R''(n) = -30 \Rightarrow R''(20) = -30 < 0$. By the second derivative test, $R$ has a local maximum at $n = 20$, which is an absolute maximum since it is the only critical number.

The best fare to maximize the revenue is then: $\$500 - 5(20) = 400$, with $180 + 3(20) = 240$ passengers and a revenue of $\$96,000$. 

**Solution of Problem 4:**