

**Test 3:** Math 214 - Differential Equations - Fall 2002 Dr. Yahdi

To receive credit for your answers you must show all your work, explain your reasoning carefully and clearly, and include all steps necessary to completely justify each answer. Any variables you use must be clearly identified. Box in your answers when it is possible. Each problem worths 20 points. Good luck!

**Problem 1.** Determine whether each of the following statement is TRUE or FALSE for any linear system (of two variables). Justify your answers to receive credit.

1. The only equilibrium point is (0,0).
2. If the corresponding matrix has a negative eigenvalue, then (0,0) is a sink.
3. If the imaginary part of an eigenvalue is not zero, there are no straight-line solutions.
4. If a solution  $Y$  satisfies:  $Y(0) \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ , then  $Y(t) \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix}$  for all  $t$ .

**Problem 2.** Consider the system:  $\frac{dY}{dt} = \begin{pmatrix} -3 & -1 \\ 1 & -1 \end{pmatrix} Y$ , where  $Y(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$ .

1. Find the equilibrium points, their type, the eigenvalues and the eigenvectors.
2. Sketch the phase portrait with some representative solutions (two in each region).
3. Sketch the graph of the corresponding solutions  $x(t)$  and  $y(t)$  for the initial condition  $Y(0) = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$ .
4. Give a formula for the solution with the initial condition  $Y(0) = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$ .

**Problem 3.** The second-order equation for a spring-mass system is:  $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} - 5x = 0$

1. Write the corresponding system and determine the type of the equilibrium point.
2. Sketch the phase plane with some representative solutions curves (two in each region).
3. Give an initial condition, (other than (0,0)), for which the solution tends to (0,0).
4. For the initial condition  $x(0) = -2$  and  $x'(0) = 4$ , plot the graph of  $x(t)$  and describe the motion of the mass.

**Problem 4.** Consider the family of differential systems depending on the parameter  $a$ :  $\frac{dY}{dt} = \begin{pmatrix} 0 & a^2 \\ -a & a \end{pmatrix} Y$

1. Sketch its corresponding curve in the Trace-Determinant plane.
2. Find all the critical locations and all the bifurcation values of  $a$ .
3. In a brief essay, discuss all different types of behaviors of the system as  $a$  vary.

**Problem 5.** Consider the system: 
$$\begin{cases} \frac{dx}{dt} = (x - y^2)x \\ \frac{dy}{dt} = (2 - y - x)y \end{cases}$$

1. Find and sketch the nullclines and the equilibrium points for this system.
2. Insert direction field "arrows" on the nullclines.
3. By linearization, give the type of the system near each equilibrium point.
4. Describe the possible behaviors of solution curve corresponding to each of the following initial conditions:

$$Y_1(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad Y_2(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad Y_3(0) = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$