Name : ____________________

Test 1: **Math 214 - Differential Equations - Fall 2002** Dr. Yahdi

To receive credit for your answers you must show all your work, explain your reasoning carefully and clearly, and include all steps necessary to completely justify each answer. Any variables you use must be clearly identified. Box in your answers when it is possible. Good luck!

Problem 1. Find the solution (explicit if possible) and its domain of definition for the initial value problem:

$$\frac{dy}{dt} = \frac{1}{t(y+1) - 2(y+1)} \qquad \text{with} \quad y(0) = 0$$

Problem 2. For each slope field **(a)-(d)** shown below, identify which of the differential equations **(1)-(6)** is appropriate. **Explain** each of your answers to receive credit.

(1) $\frac{dy}{dt} = y\cos(\pi t)$ **(2)** $\frac{dy}{dt} = t - y^2$ **(3)** $\frac{dy}{dt} = 1 - y^2$

(4) $\frac{dy}{dt} = t\cos(\pi y)$ **(5)** $\frac{dy}{dt} = \cos(\pi y)$ **(6)** $\frac{dy}{dt} = y^2 - 1$

Problem 3. Consider the differential equation $\frac{dy}{dt} = f(y)$, where the graph of $f(y)$ is given below.

1. Sketch the phase line that corresponds to this differential equation, and classify each equilibrium point as sink, source, or node
2. Describe the long-term behavior (that is to say $\lim_{t\to\infty} y(t)$) of each of the solutions y_1, y_2 and y_{13} satisfying the initial conditions:

$$y_1(0) = -1 \qquad y_2(1) = 1 \qquad y_3(2) = 1$$

3. If $y(t)$ is a solution with $y(0) = \alpha$, find all the possible values of α such that $\lim_{t\to\infty} y(t) = 1$.

Problem 4. Suppose the following differential equation is a model for a chemical reaction:

$$\frac{dy}{dt} = 2y^4 - 3y^3 - 5y^2$$

In the same system of axes, sketch the graphs of the particular solutions y_1, y_2, y_3, y_4 of this differential equation satisfying the initial conditions (don't forget to label the axes! You are NOT asked to find formulas for the solutions):

$$y_1(0) = 1, \quad y_2(0) = 3, \quad y_3(-2) = -0.5, \quad y_4(1) = -2$$

Problem 5. A **120** gallon tank initially contains **60 lbs** of salt dissolved in **100** gallons of water. Salty water containing **3 lbs/gallon** of salt flows into the tank at a rate of **3 gallons/min**, and well stirred mixture flows out at a rate of **2 gallons/min**

Write down (without solving it) the **initial value problem** for the amount of salt in the tank, (specify the variables and explain your reasoning).

Problem 6. The population **P** of a species of fish in a protected lake increases or decreases according to a logistic population model with growth-rate $k = 2$ and carrying capacity $N = 100$. It is decided that fishing will be allowed with a license, but it is unclear how many fishing licences should be issued. Suppose the average catch of a fisherman with a license is 3 fish per year (these are hard fish to catch).

1. If μ licenses are issued. Show that the adjusted model for this fish population is:

$$\frac{dP}{dt} = 2P - \frac{P^2}{50} - 3\mu$$

2. Locate the bifurcation values for μ explicitly. Draw the bifurcation diagram with several representations of the phase lines for values of the parameter mu around the bifurcation values (just one value to the left and one to the right of the bifurcation(s)). Each phase line should have direction arrows.
3. What is the largest number of licences that can be issued if the fish are to have a chance to survive in the lake? Explain!