

Practice-TEST 1: Math 214 - Differential Equations Dr. Yahdi

To receive credit for your answers you must show all your work, explain your reasoning carefully and clearly, and include all steps necessary to completely justify each answer. Any variables you use must be clearly identified. Box in your answers when it is possible. Good luck!

Problem 1. A large vat initially contains 50 gallons of sugar-water containing 10 pounds of sugar. Suppose sugar-water containing 0.5 pounds of sugar per gallon is pumped into the vat at a rate of 4 gallons per minute, while a well-mixed solution leaves the bottom of the tank at a rate of 4 gallons per minute.

a) Write down (without solving it) the **initial value problem** for the amount of sugar in the vat, (specify the variables and explain your reasoning).

b) Give an estimate for the amount of sugar in the vat after a long period of time.

Problem 2. Find the solution (explicit if possible) and its domain of definition for each initial value problem.

a) $\frac{dy}{dt} = -\frac{y}{t+5} + t$ with $y(0) = 1$

b) $\frac{dy}{dt} = \frac{-t}{y-2}$ with $y(0) = -2$

Problem 3. Suppose the following differential equation is a model for a chemical reaction:

$$\frac{dy}{dt} = 2y^4 - 3y^3 - 5y^2$$

a) Sketch the phase line for this equation, and classify each equilibrium point as a node, source, or sink.

b) In the same system of axes, sketch the graphs of the particular solutions y_1, y_2, y_3, y_4 of this differential equation satisfying the initial conditions (don't forget to label the axes!):

$$y_1(0) = 1, \quad y_2(0) = 3, \quad y_3(-2) = -0.5, \quad y_4(1) = -2$$

c) Describe the long-term behavior of each of the particular solutions above; i.e. $\lim_{t \rightarrow +\infty} y(t)$.

Problem 4. Consider the differential equation $\frac{dy}{dt} = f(t)$, where the graph of $f(t)$ is given below (use the graph in problem # 13 at the page 49 of the textbook. You can also use any arbitrary graph of $f(t)$ that you want).

a) Give a rough sketch of the slope field that corresponds to this differential equation.

b) Using this slope field, sketch the graphs of the two particular solutions with initial conditions: $y_1(0) = 0$ and $y_2(0) = 1$.

c) Describe the long-term behavior of each of these particular solutions.

d) Suppose that $y_1(1) = 1$. Explain why it is not possible to have $y_2(1) = 3$.

Problem 5. Consider the family of differential equations: $\frac{dy}{dt} = y^2 + 3y + a$ with parameter a . Locate the bifurcation values for a explicitly. Draw the bifurcation diagram with several representations of the phase lines for values of the parameter a near the bifurcation values. (Each phase line should have direction arrows.)

Problem 6. Consider the initial value problem: $\frac{dy}{dt} = y^4 + y^2 t^2$ with $y(0) = -\sqrt{2}$.

a) Use Euler's method (to two decimal places) to approximate, in 2 steps ($\Delta t = 0.5$), the value of $y(1)$, where $y(t)$ is the solution of this initial value problem.

b) Is Euler's method doing a good job in this case? Explain your answer.

MORE PROBLEMS: Problems #7-10 and #15 from Section 1.3, and problems # 32 and # 36 from section 1.6.