Summary of some Calculus I Formulas - Yahdi

Equations for lines

Equation of the Tangent line $y = f'(a)(x-a) + f(a)$
Equation of a line with slope m through (a,b): $y = m(x-a) + b$

Quadratic Formula

$$ax^2 + bx + c = 0 \Longleftrightarrow x = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \text{ or } \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

Some Algebraic Formulas:

$\sqrt[n]{A} = A^{\frac{1}{n}}$ $\quad \frac{1}{A^n} = A^{-n}$ $\quad A^n A^m = A^{n+m}$ $\quad \frac{A^n}{A^m} = A^{n-m}$

$\ln A^B = B \ln A$ $\quad \ln AB = \ln A + \ln B$

Formulas related to Economics:

Cost function: $C(x)$ **Average Cost:** $\overline{C}(x) = \frac{C(x)}{x}$

Revenue = (Price per unit) × (Quantity)

Profit: $\mathbb{P}(x) = R(x) - C(x)$

Geometric Formulas: ***C****=circumference,* ***A****=area &* ***V****= volume*

Circle: $C = 2\pi R$, $A = \pi R^2$
Sphere: $A = 4\pi r^2$, $V = \frac{4}{3}\pi R^3$
Cylinder: Area of the sides $A = 2\pi Rh$, $V = \pi R^2 h$
Triangle: Area= $\frac{1}{2}$ base × height

Gravitational attraction

$g = -9.8m/s^2 = -32ft/s^2$

Derivative using the limit:

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

L'Hospital's Rule:

$$\lim_{x \to a} \frac{f(x)}{g(x)} = I.F. \Longrightarrow \lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

Using the derivatives:

Second Derivative Test: if $f'(p) = 0$ and

- **Local maximum** for f at $p \Longleftrightarrow f''(p) < 0$ (**negative**).
- **Local minimum** for f at $p \Longleftrightarrow f''(p) > 0$ (**positive**)

What does f' say about f?

- The first **derivative** f' is **positive (+)** $\Longleftrightarrow$ f is **increasing** ($\nearrow$)
- The first **derivative** f' is **negative (−)** $\Longleftrightarrow$ f is **decreasing** ($\searrow$)

Relationships between f, f' and f''

- f **concave up** $\cup \Longleftrightarrow f'$ **increasing** $\nearrow \Longleftrightarrow f''$ **positive**.
- f **concave down** $\cap \Longleftrightarrow f'$ **decreasing** $\searrow \Longleftrightarrow f''$ **negative**.

Formulas Related to Differentiation:

Definition: $f'(x) = lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

Tangent line: $y = f'(a)(x-a) + f(a)$

sum rule: $(f+g)' = f' + g'$

Multiply by constant: $(cf)' = c(f')$

Product rule: $(fg)' = f'g + fg'$

Quotient rule: $(\frac{f}{g})' = \frac{f'g-fg'}{g^2}$

Chain rule: $(f[g(x)])' = f'[g(x)] \cdot g'(x)$

Basic Derivatives and Chain Rules short-cuts:

Power rules: $(x^n)' = n\ x^{(n-1)}$ $(u^n)' = n\ u^{(n-1)} \cdot u'$

Exponential rules: $(e^x)' = e^x$ $(e^u)' = e^u \cdot u'$

$(a^x)' = \ln a \cdot a^x$ $(a^u)' = \ln a \cdot a^u \cdot u'$

Logarithmic rules: $(\ln x)' = \frac{1}{x}$ $(\ln u)' = \frac{u'}{u}$

Trig. rules: $[\sin(x)]' = \cos(x)$ $[\sin(u)]' = \cos(u) \cdot u'$

$[\cos(x)]' = -\sin(x)$ $[\cos(u)]' = -\sin(u) \cdot u'$

$[\tan(x)]' = \frac{1}{\cos^2(x)} = \sec^2(x)$ $[\tan(u)]' = \frac{u'}{\cos^2(u)} = u'\sec^2(u)$

Inverse Trig: $[\arcsin(x)]' = \frac{1}{\sqrt{1-x^2}}$ $[\arcsin(u)]' = \frac{u'}{\sqrt{1-u^2}}$

$[\arccos(x)]' = \frac{-1'}{\sqrt{1-x^2}}$ $[\arccos(u)]' = \frac{-u'}{\sqrt{1-u^2}}$

$[\arctan(x)]' = \frac{1}{1+x^2}$ $[\arctan(u)]' = \frac{u'}{1+u^2}$

Formulas Related to antiderivatives:

Definition: $F(x) = \int f(x)\ dx =$ antiderivative of f with respect to x.

sum rule: $\int(f+g)\ dx = \int f\ dx + \int g\ dx$

Multiply by constant: $\int kf\ dx = k\int f\ dx$

Power rules for $\mathbf{n \neq -1}$: $\int x^n\ dx = \frac{x^{n+1}}{n+1} + c$

Logarithmic rules: $\int \frac{1}{x}\ dx = \ln|x| + c$

Exponential rules: $\int e^x dx = e^x + c$ $\int a^x dx = \frac{1}{\ln a}a^x + c$

Trig. rules: $\int \cos x dx = \sin x + c$ $\int \sin x dx = -\cos x + c$

$\int \sec^2 x dx = \int \frac{1}{\cos^2 x}dx = \tan x + c$

Inverse Trig: $\int \frac{1}{\sqrt{1-x^2}}dx = \arcsin x + c$ $\int \frac{1}{1+x^2}dx = \arctan x + c$

Fundamental Theorem of Calculus:

$\int_a^b f(t)\ dt = [F(t)]_a^b = F(b) - F(a)$ where F is antiderivative of f.

If $F(x) = \int_a^x f(t)\ dt$ then $F'(x) = \frac{dF(x)}{dx} = f(x)$.