Asymptotic Notation
**O-notation**

\[ O(g(n)) = \{ f(n) : \text{there exist positive constants } c \text{ and } n_0 \text{ such that } 0 \leq f(n) \leq cg(n) \text{ for all } n \geq n_0 \} . \]

\( g(n) \) is an *asymptotic upper bound* for \( f(n) \).

If \( f(n) \in O(g(n)) \), we write \( f(n) = O(g(n)) \).
Example: \(2n^2 = O(n^3)\), with \(c = 1\) and \(n_0 = 2\).

Examples of functions in \(O(n^2)\):

\[
\begin{align*}
n^2 \\
n^2 + n \\
n^2 + 1000n \\
1000n^2 + 1000n
\end{align*}
\]

Also,

\[
\begin{align*}
n \\
n/1000 \\
n^{1.99999} \\
n^2/\log \log \log n
\end{align*}
\]
$\Omega$-notation

$\Omega(g(n)) = \{ f(n) : \text{there exist positive constants } c \text{ and } n_0 \text{ such that } 0 \leq cg(n) \leq f(n) \text{ for all } n \geq n_0 \}$.

$g(n)$ is an asymptotic lower bound for $f(n)$. 
Example: $\sqrt{n} = \Omega(\lg n)$, with $c = 1$ and $n_0 = 16$.

Examples of functions in $\Omega(n^2)$:

\begin{align*}
  n^2 \\
  n^2 + n \\
  n^2 - n \\
  1000n^2 + 1000n \\
  1000n^2 - 1000n \\
  Also, \\
  n^3 \\
  n^{2.00001} \\
  n^2 \lg \lg \lg n \\
  2^{2^n}
\end{align*}
\( \Theta(g(n)) = \{ f(n) : \text{there exist positive constants } c_1, c_2, \text{ and } n_0 \text{ such that } 0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n) \text{ for all } n \geq n_0 \} \).

\( g(n) \) is an asymptotically tight bound for \( f(n) \).

**Example:** \( n^2/2 - 2n = \Theta(n^2) \), with \( c_1 = 1/4, c_2 = 1/2, \) and \( n_0 = 8 \).

**Theorem**

\( f(n) = \Theta(g(n)) \) if and only if \( f = O(g(n)) \) and \( f = \Omega(g(n)) \).
A way to compare “sizes” of functions:

\[ O \supseteq \Omega \supseteq \Theta \supseteq o \supseteq o \supseteq o \]